AD-A049 295 NEW YORK UNIV N Y COURANT INST OF MATHEMATICAL SCIENCES F/G 20/14
NONLINEAR PHENOMENA IN ELECTROMAGNETIC THEORY AND ACOUSTICS.(U)
JAN 78 J B KELLER, F C HOPPENSTEADT DAAG29-74-G-0219
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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: 19 12387.4-M
2. GOVT ACCESSION NO.: 18 ARO
3. RECIPIENT'S CATALOG NUMBER: 9
4. TITLE (and Subtitle): 6 Nonlinear Phenomena in Electromagnetic Theory and Acoustics.
5. TYPE OF REPORT & PERIOD COVERED: Final Report, 1 Jun 74 - 30 Sep 77,
6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): 10 Joseph B. Keller, Frank C. Hoppensteadt
8. CONTRACT OR GRANT NUMBER(s): 15 DAAG29-74-G-0219
9. PERFORMING ORGANIZATION NAME AND ADDRESS: New York University, Courant Institute of Mathematical Sciences, New York, New York 10012
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: U. S. Army Research Office, Post Office Box 12211, Research Triangle Park, NC 27709
12. REPORT DATE: 11 Jan 78
13. NUMBER OF PAGES: 21
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 12 28p.
15. SECURITY CLASS. (of this report): Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

12

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

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FEB 1 1978
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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Electromagnetic wave propagation; nonlinear circuit theory; stochastic processes; random media; wave guides; sound transmission; tubes; vibration; perturbation theory; nonlinear equations; oscillations; differential equations; thermal convection; difference equations; integral equations; stability; boundary value problems; bifurcation theory; asymptotic series; continuum mechanics

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The research projects supported by these grants include investigations into problems of nonlinear wave propagation, nonlinear electrical circuits, propagation in wave guides, thermal convection and continuum mechanics. The success of these projects rested on our unified and coherent development and applications of multi-scale perturbation methods to study the complicated nonlinear systems which describe these physical phenomena. The research projects are described in forty-seven research papers and books. These are summarized in this report by area of application.

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I. Reports published and accepted for publication.

The research projects supported entirely or in part by this grant are described in forty-seven research papers and books. These are summarized below by area of application.

A. Electromagnetic phenomena.

1. Electromagnetic wave propagation.

ARO-10, "Perturbation Theory of Nonlinear Electromagnetic Wave Propagation" by Joseph B. Keller and Martin H. Millman, The Physical Review, 181, 5, 1730-1747 (1969).

A perturbation method, previously employed to treat nonlinear boundary-value problems involving partial differential equations, is used to study nonlinear electromagnetic wave propagation. The problems considered are I. electromagnetic wave propagation in a waveguide containing a nonlinear isotropic medium; II. electromagnetic oscillations in a cavity containing a nonlinear dispersive anisotropic medium; III. propagation and interaction of electromagnetic waves in a nonlinear dispersive anisotropic medium; IV. reflection of an electromagnetic wave from a nonlinear dispersive uniaxial medium. The method avoids secular terms, which arise in some perturbation treatments. The results show how the wave number, propagation velocity, and angle of refraction of a wave depend upon its amplitude.

ARO-47, "Dynamics of the Josephson Junction" by Frank C. Hoppensteadt, Willard L. Miranker and Mark Levi, Quart. Jour. on Appl. Math., in press.

2. Nonlinear circuit theory.

ARO-35, "Asymptotic Stability in Networks with Parasites" by Frank Hoppensteadt, Proceed. 11th Annual Allerton Conf. on Circuit and System Theory, 703-707 (1973).

An important question is how the presence of parasitics can influence the large-time stability of an electrical network. Many investigations of this problem have been based on the construction of Liapunov functions, a method which can become quite tedious. However, frequently much more information can be obtained under the same conditions by using a well known perturbation method to construct approximate solutions of the problem. These lead to useful approximations to the domain of stability, to the large-time state of the system, and in asymptotically steady problems, to the system's steady state.

ARO-36, "Numerical Solution of Differential Equations with Rapidly Changing Solutions" by Frank Hoppensteadt and Willard L. Miranker, IBM Tech. Report RC4792, 1-39 (1974).

Initial value problems for weakly nonlinear systems of ordinary differential equations are studied here. First a perturbation result is obtained which is shown to give a valid approximation to the solution. The leading term in the approximation expansion is determined by solving two associated problems. This approximation is particularly relevant to problems which have rapidly oscillating and rapidly decaying solutions. Next, a numerical procedure, based on the asymptotic approximation, is formulated. The procedure bypasses a major technical problem in the construction of the approximation, and it provides an attractive method for computing the solution.

ARO-45, "Frequency Entrainment of a Forced Van der Pol Oscillator" by J. E. Flaherty and F. C. Hoppensteadt, Studies in Appl. Math., in press.

A van der Pol relaxation oscillator that is subjected to external sinusoidal forcing can exhibit stable and unstable periodic and almost periodic responses. For some forcing amplitudes it even happens that two stable subharmonic having different periods may coexist. We investigate here the stable responses of such forced oscillators. By numerically computing the rotation number of stable oscillations for various values of the forcing amplitude and oscillator tuning, we obtain descriptions of regions of phase locking, successive bifurcation of stable subharmonic and almost periodic oscillations, and overlap regions where two distinct stable oscillations can coexist.

3. Propagation in random media.

ARO-18, "Stochastic Differential Equations with Applications to Random Harmonic Oscillators and Wave Propagation in Random Media" by George Papanicolaou and Joseph B. Keller, SIAM J. Appl. Math., 21, 2, 287-305 (1971).

The two-time method is used to obtain an expansion, valid for ε small and t large, of the vector solution $u(t, \varepsilon)$ of an abstract ordinary differential equation involving ε. The same method is used to get expansions of functions of u. The results are shown to apply to the solutions of stochastic equations. They are used to find the first two moments and the transition probability of the displacement of a harmonic oscillator with spring constant a random function of t. The result contains the condition for mean square stability due to Stratonovich. The results are also applied to one-dimensional wave propagation through a layer with refractive index a random function of position. They are used to find the mean square amplitude reflection and transmission coefficients, which are just the mean power reflection and transmission coefficients. A graph of the mean square transmission coefficient as a function of layer thickness is presented. The results are also compared with those obtainable by other methods.

B. Acoustics.

1. Sound propagation in wave guides.

ARO-13, "Finite-Amplitude Sound-Wave Propagation in a Waveguide" by Joseph B. Keller and Martin H. Millman, J. Acoust. Soc. Am., 49, 1 (Part 2), 329-333 (1971).

A sound wave with angular frequency ω and finite amplitude ϵ, propagating in a rigid waveguide of any non-rectangular cross section, is considered. The pressure and velocity potential of the wave, corresponding to each mode of the linear theory, are determined as power series in ϵ. The leading term is just the mode given by the linear theory and further terms are finite-amplitude corrections to it. The wavenumber or propagation constant k_n of the nth mode is found to depend upon ϵ and is also determined as a power series. The fact that k_n depends upon ϵ, and the expression for $k_n(\epsilon)$, is the most interesting consequence of the analysis. This shows that the propagation velocity ω/k_n depends upon ϵ. The results are specialized to a waveguide of circular cross section. Then numerical values of $k_n(\epsilon)$ are given for a circular guide filled with air for $n=1, 2, 3, 4$, and 5. The lowest mode ($n=0$) is not considered because shocks occur in it. The method of analysis is a perturbation expansion adapted to eliminate secular terms. It has been used before to treat periodic finite-amplitude sound waves in a closed container and periodic finite-amplitude vibrations of strings and beams by J. B. Keller and L. Ting [Comm. Pure Appl. Math. 19, 371 (1966)], and to treat nonlinear electromagnetic wave propagation [Phys. Rev. 181, 1730 (1969)] and other nonlinear boundary-value problems by the authors [J. Math. Phys. 10, 342 (1969)].

ARO-32, "Nonlinear Forced and Free Vibrations in Acoustic Waveguides" by Joseph B. Keller, J. Acoust. Soc. Am., 55, 3, 524-527 (1974).

Nonlinear acoustic theory is used to show that the cutoff frequencies and the resonant frequencies of modes in acoustic waveguides of finite length depend upon the mode amplitude. The amplitude ϵ_n of mode $n>0$, produced by a periodic piston motion of amplitude δ_n, is also determined. It is found that at and near a resonant frequency, $\epsilon_n \sim \delta_n^{1/3}$ while at and near the cutoff frequency $\epsilon_n \sim \delta_n^{1/2}$. Away from these frequencies $\epsilon_n \sim \delta_n$, as linear theory predicts. Thus the infinite amplitude, which linear theory yields at cutoff and resonance, is avoided. The results are obtained simply by using the result for the propagation constant as a function of mode amplitude given by J. B. Keller and M. H. Millman [J. Acoust. Soc. Am. 49, 329–333 (1971)].

2. Acoustic vibrations in tubes.

ARO-28, "Nonlinear Resonant Oscillations in Open Tubes" by Brian R. Seymour and Michael P. Mortell, J. Fluid Mech., 60, 4, 733-749 (1973).

A gas in a tube, one end of which is open, is driven by a periodic applied velocity or pressure at or near a resonant frequency. Damping is introduced into the system by radiation of energy through the open end. It is shown that shocks are possible at an open end and that there is a critical level of damping which ensures a continuous gas response for all frequencies. At the critical level the amplitude of the response is $O(\epsilon^{\frac{1}{2}})$, where ϵ is the amplitude of the input, and it is bounded by the amplitude predicted by linear theory. There is agreement with the qualitative experimental results available.

ARO-30, "Standing Waves in an Open Pipe: A Nonlinear Initial-Boundary Value Problem" by Michael P. Mortell, J. Appl. Math. and Phys., 24, 473-487 (1973).

ARO-33, "Nonlinear Geometrical Acoustics" by Brian S. Seymour, in Mechanics Today, Vol. 2, ed. by S. Nemat-Nasser, 1975.

ARO-16, "Self-Excited Thermo-Acoustic Vibrations in the Rijke Tube by J. B. Keller and M. H. Millman, submitted.

C. Perturbation theory.

1. Nonlinear oscillations.

a. Nonlinear wave equation.

ARO-12, "Asymptotic Solutions of Initial Value Problems for Nonlinear Partial Differential Equations" by Joseph B. Keller and Stanley Kogelman, SIAM J. Appl. Math., 18, 4, 748-758 (1970).

ARO-39, "Periodic Solutions of Nonlinear Wave Equations in N-Dimensional Space" by L. Ting, SIAM J. Appl. Math., in press. The solvability conditions for a linear inhomogeneous wave equation with solutions periodic in space and time are established as integral conditions. These conditions are incorporated in a systematic perturbation theory for the determination of the periodic solutions of the nonlinear wave equation $u_{tt} - \Delta u = \varepsilon f(u)$ with ε as the small parameter. Applications to boundary value problems are presented.

ARO-44, "Slowly Modulated Oscillations in Nonlinear Diffusion Processes" by Donald S. Cohen, Frank C. Hoppensteadt and Robert M. Miura, SIAM J. Appl. Math., 33, 2, 217-229 (1977).

It is shown here that certain systems of nonlinear (parabolic) reaction-diffusion equations have solutions which are approximated by oscillatory functions in the form $R(\xi - c\tau)P(t^*)$ where $P(t^*)$ represents a sinusoidal oscillation on a fast time scale t^* and $R(\xi - c\tau)$ represents a slowly-varying modulating amplitude on slow space (ξ) and slow time (τ) scales. Such solutions describe phenomena in chemical reactors, chemical and biological reactions, and in other media where a stable oscillation at each point (or site) undergoes a slow amplitude change due to diffusion.

b. Nonlinear ordinary differential equations.

ARO-1, "Periodic Vibrations of Systems Governed by Nonlinear Partial Differential Equations" by Joseph B. Keller and Lu Ting, Comm. Pure Appl. Math., 19, 4, 371-420 (1966).

ARO-41, "Iterated Averaging Methods for Systems of Ordinary Differential Equations with a Small Parameter" by S. Persek and F. C. Hoppensteadt, Comm. Pure Appl. Math., in press. Extensions of Bogoliuboff's averaging methods are derived by multi-time perturbation procedures. These procedures are shown to provide accurate approximate solutions in cases where the averaged system has stationary and decaying modes. Estimates of the approximations are found on large $O(1/\varepsilon^2)$ and infinite time intervals, depending on stability properties of certain second order averaged systems. Problems not amenable to Bogoliuboff's methods, such as the construction of periodic solutions and their transients near Hopf bifurcations in quadratic systems, can be solved by this iterated averaging method.

c. <u>Thermal convection</u>.

ARO-2, "Periodic Oscillations in a Model of Thermal Convection" by Joseph B. Keller, J. Fluid Mech., 26, 3, 599-606 (1966).

Periodic oscillations are found in a one-dimensional model of thermal convection. The model consists of a fluid-filled tube bent into rectangular shape and standing in a vertical plane. The fluid is heated at the centre of the lower horizontal segment and cooled at the centre of the upper horizontal segment. When a certain parameter exceeds unity, a periodic motion of the fluid is found in which the flow is always in the same direction but in which the speed varies. Inertia is unimportant for this oscillation, which depends upon the interplay between frictional and buoyancy forces.

d. Nonlinear difference and integral equations.

ARO-40, "A Nonlinear Renewal Equation with Periodic and Chaotic Solutions" by Frank C. Hoppensteadt, SIAM-AMS Proceedings, 10, 51-60 (1976).

A nonlinear renewal equation which arises in several areas of mathematical population theory is studied by a combination of mathematical and numerical analysis. The model is characterized by two parameters: m, a measure of the population's viability and fertility, and μ, the (normalized) length of the population's reproductive window. Solutions are described for all values of these parameters, $0 \leq m \leq 4$, $0 \leq \mu \leq 1$, by a combination of multi-time perturbation analysis and numerical solution. In various regions, the solutions are shown to be described by Burgers' equation, a Korteweg-deVries equation and by a nonlinear difference equation. Numerical methods are used to investigate the remaining regions.

2. Nonlinear wave propagation.

ARO-10, "Perturbation Theory of Nonlinear Electromagnetic Wave Propagation" by Joseph B. Keller and Martin H. Millman, The Physical Review, 181, 5, 1730-1747 (1969).

A perturbation method, previously employed to treat nonlinear boundary-value problems involving partial differential equations, is used to study nonlinear electromagnetic wave propagation. The problems considered are I. electromagnetic wave propagation in a waveguide containing a nonlinear isotropic medium; II. electromagnetic oscillations in a cavity containing a nonlinear dispersive anisotropic medium; III. propagation and interaction of electromagnetic waves in a nonlinear dispersive anisotropic medium; IV. reflection of an electromagnetic wave from a nonlinear dispersive uniaxial medium. The method avoids secular terms, which arise in some perturbation treatments. The results show how the wave number, propagation velocity, and angle of refraction of a wave depend upon its amplitude.

ARO-24, "Asymptotic Theory of Nonlinear Wave Propagation" by Stanley Kogelman and Joseph B. Keller, SIAM J. Appl. Math., 24, 3, 352-361 (1973).

By extending the asymptotic method of G. E. Kuzmak, J. C. Luke has shown how G. B. Whitham's theory of nonlinear wave propagation can be derived directly from the partial differential equation without using the variational principle, in special cases. We apply the same method to a more general class of nonlinear second order partial differential equations or systems of first order equations containing a small parameter ε and obtain asymptotic expansions of the solutions.

ARO-3, "A Modified Stationary Principle for Nonlinear Waves" by Frederic Bisshopp, J. Diff. Eqns., 5, 3, 592-605 (1969).

Two theorems related to propagation of modulated, nonlinear waves which are governed by stationarity of the integral of a Lagrangian density are derived. They are then used in the analysis of asymptotic approximations which represent nearly uniform wave trains. A relatively detailed treatment of a simple case is followed by a summary of some results which apply to the propagation of capillary-gravity waves in a liquid.

3. Nonlinear stability theory.

ARO-6, "Finite Amplitude Effect on the Stability of a Jet of Circular Cross-Section" by D. P. Wang, J. Fluid Mech., 34, 2, 299-313 (1968).

The effect of finite amplitude on the stable and unstable states of a column of an ideal fluid of circular cross-section under the action of surface tension is studied. The method of solution is a formal extension of the linearized theory; it consists of assuming that the exact solution may be expanded in a power series of a small parameter characterizing the amplitude. The calculation is carried out to the point where the first non-trivial term of the finite amplitude effect is obtained. For the stable states, the result shows that the characteristic wavelength of a disturbance which appears to be stationary with respect to an observer is decreased by the finite amplitude effect. For the unstable states, it reveals that the growth rate depends not only on the wavelength and the magnitude but also on the type of disturbance imposed initially. The last result is a direct consequence of the fact that two independent types of initial disturbance, the disturbance of the velocity field and the disturbance of the free surface, may be imposed simultaneously on the jet.

ARO-25, "Multiple Scaling Techniques for Nonlinear Stability Problems" by S. Kogelman, Proc. Conf. on Stability Theory, Washington State Univ., 179-194 (1972).

We consider a model equation which preserves the basic features of some typical nonlinear hydrodynamic stability problems. By introducing two time scales we are able to obtain asymptotic solutions to the initial value problem. Furthermore, introduction of an additional spatial variable enables us to account for modes whose wave numbers are very near the critical wave number predicted by linear theory.

ARO-14, "Transient Behavior of Unstable Nonlinear Systems with Applications to the Benard and Taylor Problems" by Stanley Kogelman and Joseph B. Keller, SIAM J. Appl. Math., 20, 4, 619-637 (1971).

A nonlinear initial value problem containing two parameters. ε and λ, is considered for a function $u(t)$. For each t, $u(t)$ is a vector in a Hilbert space. When $\varepsilon = 0$ the problem has a steady state solution u_0 for any value of λ. This solution is assumed to be linearly stable for $\lambda < \lambda_c$ and linearly unstable for $\lambda > \lambda_c$ for some value λ_c. This means that the linear problem for the derivative $u_t(t)$ at $\varepsilon = 0$ has exponentially growing solutions for $\lambda > \lambda_c$ but not for $\lambda < \lambda_c$. The solution $u(t)$ is found for ε small and λ slightly larger than λ_c. It is found that u contains one unstable mode which initially increases exponentially but ultimately approaches a constant, while all other modes decay. up to terms of order ε^2. To this order $u(t)$ approaches a steady state solution $u(\infty)$ different from u_0. as t increases. The method of analysis is the two-time perturbation method. The method and results are illustrated by applying them to the Bénard problem of convective heat transfer through a horizontal layer of viscous fluid heated from below, and the Taylor problem of the instability of the Couette flow of a viscous fluid between coaxial rotating circular cylinders.

ARO-27, "Nonlinear Stability Theory" by Joseph B. Keller, Proceed. Conference on Stability Theory, Washington State University, 85-98 (1972).

ARO-37, "Nonlinear Stability Analysis of Static States which Arise Through Bifurcation" by F. Hoppenstead and N. Gordon, Comm. Pure Appl. Math., 28, 355-373 (1975).

Multi-time expansion solutions are constructed for systems of nonlinear partial differential equations near bifurcated states. The class of problems studied includes several problems of interest in stability of fluid flow, such as the Benard and Taylor problems, problems of nonlinear diffusion, as well as many initial value problems for systems of ordinary differential equations. The main result gives a rigorous derivation of an expansion for the solution as a sum of expansions for the bifurcated state and one each for slow and fast time transients to that state. The expansion for the transient gives an approximation to the domains of attraction of stable bifurcated states. Also, as a consequence of the method, we give a rigorous derivation of the Landau equation for these systems.

4. Perturbation theory.

ARO-29, "On the Asymptotic Solution of a Two-Parameter Boundary Value Problem of Chemical Reactor Theory" by Jye Chen and R. E. O'Malley, Jr., SIAM J. Appl. Math., 26, 4, 717-729 (1974).

The paper provides an asymptotic solution to boundary value problems of the form:

$$\varepsilon_1 y'' + a(x)y' = f(x, y, z), \qquad \varepsilon_1\varepsilon_2 z'' + b(x)z' = g(x, y, z),$$

$$y'(0) = z'(0) = 0, \quad \varepsilon_1 y'(1) + y(1) = A, \quad \varepsilon_1\varepsilon_2 z'(1) + z(1) = B,$$

where ε_1 and ε_2 are small positive parameters simultaneously tending to zero, a and b are strictly positive, and a, b, f, and g are infinitely differentiable in their arguments.

ARO-34, "Asymptotic Stability in Singular Perturbation Problems. II: Problems Having Matched Asymptotic Expansion Solutions" by Frank Hoppensteadt, J. Diff. Eqs., 15, 3, 510-521 (1974).

The stability of systems of ordinary differential equations of the form

$$dx/dt = f(t, x, y, \epsilon), \qquad \epsilon\, dy/dt = g(t, x, y, \epsilon),$$

where ϵ is a real parameter near zero, is studied. It is shown that if the reduced problem

$$dx/dt = f(t, x, y, 0), \qquad 0 = g(t, x, y, 0),$$

is stable, and certain other conditions which ensure that the method of matched asymptotic expansions can be used to construct solutions are satisfied, then the full problem is asymptotically stable as $t \to \infty$, and a domain of stability is determined which is independent of ϵ. Moreover, under certain additional conditions, it is shown that the solutions of the perturbed problem have limits as $t \to \infty$. In this case, it is shown how these limits can be calculated directly from the equations

$$f(\infty, x, y, \epsilon) = 0 \qquad g(\infty, x, y, \epsilon) = 0$$

as expansions in powers of ϵ.

ARO-5, "A Non-Linear Singular Perturbation Problem Arising in the Study of Chemical Flow Reactors" by R. E. O'Malley, Jr., J. Inst. Maths. Applics., 6, 12-20 (1969).

This paper presents a method of obtaining the complete asymptotic solution of boundary value problems of the form

$$\left.\begin{aligned} \varepsilon y'' - b(x)y' - g(x,y) &= 0 \\ \varepsilon y'(0) &= y(0) - \alpha \\ y'(1) &= \beta \end{aligned}\right\}$$

for $x \in [0,1]$ where $b(x)$ is strictly positive and for ε small and positive. Physically, the problem arises in determining the steady-state concentration of a substance in a chemical flow reactor. A "two-variable" expansion procedure is used.

ARO-9, "Perturbation Theory of Nonlinear Boundary-Value Problems" by Martin H. Millman and Joseph B. Keller, J. Math. Phys., 10, 2, 342-361.

A systematic perturbation theory is presented for the analysis of nonlinear problems. The lowest-order result is just that obtained by linearizing the problem, and the higher-order terms are the solutions of inhomogeneous linear problems. The essential feature of the method is the procedure for avoiding secular terms, which is based on the Lindstedt-Poincaré technique employed in celestial mechanics. The method is applied to the following nonlinear boundary value problems: (1) temperature distribution due to a nonlinear heat source or sink; (2) self-sustained oscillations of a system with infinitely many degrees of freedom; (3) forced vibrations of a "string" with a nonlinear restoring force; (4) superconductivity in a body of arbitrary shape with external magnetic field; (5) superconductivity in an infinite film with parallel magnetic field; (6) comparison of solutions of the Hartree, Fock, and Schrödinger equations for the helium atom. The results in each case are different both qualitatively and quantitatively from those of the linear theory.

ARO-22, "Justification of Matched Asymptotic Expansion Solutions for Some Singular Perturbation Problems" by Frank Hoppensteadt, Amer. Math. Society, 23, 337-341 (1973).

Several types of problems involving non-linear partial differential equations having small parameters are described which are amenable to the method of matched asymptotic expansions. This method gives approximate solutions to the problems which are valid over large time intervals. This theory is applicable to many problems involving the Navier-Stokes equations and to quite general operator equations.

ARO-4, "A Boundary Problem for Certain Nonlinear Second Order Differential Equations with a Small Parameter" by R. O'Malley, Arch. Rat. Mech. and Anal., 29, 66-74 (1968).

ARO-11, "Asymptotic Series Solutions of Some Nonlinear Parabolic Equations with a Small Parameter" by F. Hoppensteadt, Arch. Rat. Mech. and Anal., 35, 284-298 (1969).

ARO-17, Bifurcation Theory and Nonlinear Eigenvalue Problem, W. A. Benjamin Co., New York (1969).

ARO-38, "Differential Equations Having Rapidly Changing Solutions: Analytic Methods for Weakly Nonlinear Systems" by F. C. Hoppensteadt, J. Diff. Eqns., 22, 237-249 (1976).

Initial value problems for weakly nonlinear systems of differential equations are studied here. A perturbation result is obtained which is shown to give a valid approximation to the solution. The approximation is determined by solving two associated problems, and it is especially derived for problems which have rapidly oscillating and rapidly decaying solutions.

5. Continuum mechanics, etc.

ARO-7, "Rossby Waves in the Presence of Random Currents" by Joseph B. Keller and George Veronis, J. Geophys. Res., 74, 8, 1941-1951 (1969).

The effect of weak random currents on time periodic Rossby waves is determined by a perturbation method devised previously. As a result the dispersion equation for the average Rossby wave is obtained, and from it the propagation velocity and decay rate are found. These quantities are evaluated explicitly for steady unidirectional random currents. From the results simple forms are obtained when the Rossby wavelength is either large or small in comparison to the correlation length of the current. For some directions of propagation of the wave the velocity is increased by the current, whereas for other directions it is decreased, and a physical explanation of this is presented. Random currents will scatter the waves and cause them to decay. If the current motion can add to the energy flux of the waves, the energy flux may grow drawing energy from the currents. It is found that east-west currents serve mainly to scatter the waves but north-south currents can contribute to the growth of waves. The growth phenomenon is interpreted as an instability of the currents to Rossby waves to which they can give up some of their energy. The maximum growth rate occurs for waves traveling either northwestward or southwestward, because the energy flux of these waves is in the same direction as the direction of the random currents.

ARO-20, "Flow of a Viscous Fluid Through an Elastic Tube with Applications to Blood Flow" by S. I. Rubinow and Joseph B. Keller, J. Theor. Biol., 35, 299-313 (1972).

The steady flow of a viscous fluid through an elastic tube is determined theoretically. It is assumed that Poiseuille's law holds locally and that at any location the radius of the tube is determined by the transmural pressure difference. The stress strain curve of the tube material is assumed to be known. The flux through the tube is found as a function of inlet pressure, outlet pressure, external pressure, tube length, fluid viscosity and the elastic properties of the tube. The results are displayed graphically for the human external iliac artery. They are in good agreement with experimental steady flow measurements on rubber tubes, on venous return to the heart, and on isolated cats' lungs. The effect of tube collapse is also considered.

ARO-21, "Post Buckling Behavior of Elastic Tubes and Rings with Opposite Sides in Contact" by Joseph E. Flaherty, Joseph B. Keller and S. I. Rubinow, SIAM J. Appl. Math., 23, 4, 446-455 (1972).

An elastic tube of circular cross section or an elastic ring can buckle if p, the outside pressure minus the inside pressure, exceeds the buckling pressure p_{b2}. As p increases above p_{b2} the cross section becomes somewhat elliptical. Ultimately opposite sides touch at one point at the contact pressure p_{c2}. As p increases above p_{c2} the curvature of the cross section at the points of contact decreases until it becomes zero at p_{02}. For $p > p_{02}$ contact occurs along a straight-line segment which increases in length as p increases. The pressures p_{b2}, p_{c2} and p_{02} are determined numerically, and the shape of the cross section is found for various values of p. Graphs of the results are shown. Above p_{02} a similarity solution is found. Analogous results are obtained for the nth buckling mode, $n > 2$. The flux of an incompressible viscous fluid through the buckled tube is also determined as a function of p. The results may be applicable to the collapse of veins and the flow of blood through them.

ARO-23, "Contact Problems Involving a Buckled Elastica" by Joseph E. Flaherty and Joseph B. Keller, SIAM J. Appl. Math., 24, 2, 215-225 (1973).

The classical problem of the buckling of a slender rod (an elastica) is reexamined numerically and analytically taking into account contact between different parts of the rod. The lowest mode of the clamped elastica and the third mode of the pinned elastica are treated. Other modes can be treated in a similar way. It is found that for each of these modes there is a compressive buckling load $P_b > 0$ and a contact load $P_c > P_b$. The rod remains straight when $P \leqq P_b$, the rod buckles without contact for $P_b < P < P_c$, and two points of the rod are in contact for $P \geqq P_c$. The contact points move toward the ends of the rod as P increases. In each case an asymptotic formula is obtained for the location of the contact points as $P \to \infty$. In both cases a similarity solution is found for the shape of the elastica between the two contact points. In the pinned case there is a range of loads just above P_c for which the distance between the ends of the rod increases as the load increases. This indicates instability and the possibility of "snap through."

ARO-31, "Resonant Surface Waves" by J. R. Ockendon and H. Ockendon, J. Fluid Mech., 59, 2, 397-413 (1973).

Small amplitude forced horizontal or vertical oscillations of a container of liquid with a free surface may give rise to motions in the liquid on a scale much greater than the forcing amplitude. Three such situations are analysed and, in those cases where the response is still small compared with the dimensions of the container, explicit asymptotic solutions for the liquid motion are obtained.

ARO-19, "Temperature of a Nonlinearly Radiating Semi-Infinite Solid" by Joseph B. Keller and W. E. Olmstead, Q. App. Math., 559-566 (1972).

ARO-43, "Applications of Multi-Time Methods to Pattern Recognition and Other Problems" by F. C. Hoppensteadt and W. L. Miranker, Proc. Journees Math. sur les Perturbation Singulieres (1976), Springer-Verlag, in press.

Systems of difference equations containing small parameters are studied by a constructive perturbation scheme analogous to the one developed by the authors for the study of differential equations. The method results in an averaging procedure for difference equations, and it is particularly well suited to certain highly oscillatory, non-linear systems. The method is applied to problems for population genetics, pattern recognition, regression analysis, and the numerical analysis of still differential equations.

ARO-26, "Some Trends in Applied Mechanics" by J. B. Keller, Proc. of the 1st Iranian Congress of Civil Eng. and Mech., May 5-9, 103-124 (1972).

ARO-15, "Pulse Propagation in a Laterally Heterogeneous Solid Elastic Sphere" by Z. S. Alterman, J. Aboudi and F. C. Karal, Geophys. J. R. Astro. Soc., 21, 243-260 (1970).

The motion of a solid elastic sphere caused by an impulsive point source has been calculated by a finite difference scheme. Results are obtained for a radially and laterally heterogeneous sphere. Both discontinuous changes in elastic parameters and density, and their continuous variation inside the sphere are treated. The results show reflection and diffraction effects. A stability analysis for the finite difference scheme in spherical co-ordinates is given. The method of solution presented here is general enough to include the actual variations in density and elastic parameters in the Earth. This includes the well-known models of radial distribution and the recently observed lateral inhomogeneity in the mantle.

ARO-42, "Quantitative Theories of Carcinogenesis" by Alice Whittemore and Joseph B. Keller, SIAM Review, in press.

Various mathematical theories of the genesis of cancer are presented. They involve the transformation of a normal cell and the subsequent proliferation of the transformed cell and its descendants to form a tumor. The theories predict the rate of tumor occurrence as a function of time and of the concentration of carcinogenic agents, which cause transformation and may also accelerate growth. The consequences of the theories are deduced and compared with data on human cancer incidence and with data from experiments on laboratory animals. A new theory is presented which includes features of most previous theories, but which goes beyong them in being capable of accounting for the results of initiation-promotion experiments.

ARO-46, "Peeling, Slipping and Cracking—Some One-Dimensional Free Boundary Problems in Mechanics" by R. Burridge and J. B. Keller, SIAM Review, in press.

We examine several problems involving a one-dimensional continuum with a free boundary. Apart from their intrinsic interest, they illuminate the related but far more complex problems which arise in fracture mechanics, in frictional sliding such as may occur in shallow earthquakes, and in the testing of adhesives.

First we treat peeling of adhesive tape from a surface. The tape is regarded as a flexible string, part of which adheres to the surface and part of which is separated from it. The free boundary is the point (or points) at which separation takes place and the problem is to find the trajectory of this point (or points) together with the motion of the separated section of the string. We treat both infinitesimal and finite deformations.

We also treat the related problem of the sliding of a segment of a stretched string pressed against a rough plane under the influence of transverse loads. Adhesive forces may be either present or absent at the end points of the moving segment, which form the free boundaries.

II. Summary.

The research projects supported by these grants include investigations into problems of nolinear wave propagation, nonlinear electrical circuits, propagation in wave guides, thermal convection and continuum mechanics. The success of these projects rested on our unified and coherent development and applications of multi-scale perturbation methods to study the complicated nonlinear systems which describe these physical phenomena.

ARO Reports and Publications

ARO-1	J. B. Keller Lu Ting	Periodic Vibrations of Systems Governed by Nonlinear Differential Equations Pub: Comm. Pure Appl. Math., 19, 4, (1966)
ARO-2	J. B. Keller	Periodic Oscillations in a Model of Thermal Convection Pub: J. Fluid Mechanics, 26, 3, (1966), pp. 599-606
ARO-3	F. Bisshopp	A Modified Stationary Principle for Nonlinear Waves Pub: J. Differential Equations, 5, 3, (1969) pp. 592-605
ARO-4	R. E. O'Malley	A Boundary Problem for Certain Nonlinear Second Order Differential Equations with Small Parameter Pub: Arch. Rat. Mech. Anal., 29, 1, (1968), pp. 66-74
ARO-5	R. E. O'Malley	A Nonlinear Singular Perturbation Problem Arising in the Study of Chemical Flow Reactors Pub: J. of the Institute of Math and its Applics., 6, pp. 12-20, (1969)
ARO-6	D. P. Wang	Finite Amplitude Effect on the Stability and Instability of a Jet of Circular Cross Section Pub: J. Fluid Mech's, 34, 2, (1968), pp. 299-313
ARO-7	J. B. Keller G. Veronis	Rossby Waves in the Presence of Random Currents Pub: Geophysical Research, 74, 8, (1969), pp. 1941-1951
ARO-8	J. B. Keller	Perturbation Theory Pub: Michigan State U., 1968
ARO-9	M. H. Millman J. B. Keller	Perturbation Theory of Nonlinear Boundary Value Problems Pub: J. Math. Physics, 10, 2, (1969), pp. 342-361
ARO-10	J. B. Keller M. Millman	Perturbation Theory of Nonlinear Electromagnetic Wave Propagation Pub: Physical Review, 181, 5, (1969)

ARO-11	F. Hoppensteadt	Asymptotic Series Solutions of Some Nonlinear Parabolic Equations with a Small Parameter Pub: Archive for Rational Mechanics and Analysis 35, 4, (1969) pp. 284-298
ARO-12	J. B. Keller S. Kogelman	Asymptotic Solutions of Initial Value Problems for Nonlinear Differential Equations Pub: SIAM J. Appl. Math., 18, 4, (1970), pp. 748-758.
ARO-13	J. B. Keller M. H. Millman	Finite Amplitude Sound Wave Propagation in a Waveguide Pub: J. Acoustical Society of Amer., 49, 1, ~~Part 2~~, (1971), pp. 329-333
ARO-14	J. B. Keller S. Kogelman	Transient Behavior of Unstable Nonlinear Systems with Applications to the Bénard and Taylor Problems Pub: SIAM J. Appl. Math., 20, 4, (1971), pp. 619-637.
ARO-15	F. Karal Z. Alterman J. Aboudi	Pulse Propagation in a Laterally Heterogeneous Solid Elastic Sphere Pub: Geophys. J. Astr. Soc., 21 (1970), pp. 243-260.
ARO-16	J. B. Keller M. H. Millman	Self-excited Thermo-acoustic Vibrations in the Rijke Tube Sub:
ARO-17	J. B. Keller S. Antman	Bifurcation Theory and Nonlinear Eigenvalue Problems Pub: W. A. Benjamin Co., New York, 1969.

ARO-18	J.B. Keller G. Papanicolaou	Stochastic Differential Equations with Applications to Random Harmonic Oscillators and Wave Propagation In Random Media <u>Pub</u>: SIAM J. Appl. Math., <u>21</u>, 2, (1971), pp. 287-305.
ARO-19	J.B. Keller W.E. Olmstead	Temperature of a Nonlinearly Radiating Semi-infinite Solid <u>Pub</u>: Quarterly of Appl. Math., Jan. (1972) pp. 287-305.
ARO-20	J.B. Keller S.I. Rubinow	Flow of a Viscous Fluid through an Elastic Tube with Applications to Blood Flow <u>Pub</u>: J. Theoretical Biology, <u>35</u>, (1972), pp. 299-313.
ARO-21	J.E. Flaherty J.B. Keller S.I. Rubinow	Post Buckling Behavior of Elastic Tubes and Rings With Opposite Sides in Contact <u>Pub</u>: SIAM J. Appl. Math., <u>23</u>, 4, (1972) pp. 446-455.
ARO-22	F. Hoppensteadt	Justification of Matched Asymptotic Expansion Solutions for Some Singular Perturbation Problems <u>Pub</u>: AMS Proceedings of the Symposia of Pure Math, <u>23</u>, (1973) pp. 337-341.
ARO-23	J. B. Keller J.E. Flaherty	Contact Problems Involving a Buckled Elastica <u>Pub</u>: SIAM J. Appl. Math, <u>24</u>, 2, (1973) pp. 215-225.
ARO-24	J.B. Keller S. Kogelman	Asymptotic Theory of Nonlinear Wave Propagation <u>Pub</u>: SIAM J. Appl. Math., <u>24</u>, 3, (1973) pp. 352-361.
ARO-25	Stanley Kogelman	Multiple Scaling Techniques for Nonlinear Stability Problems <u>Pub</u>: Proceed. Conference on Stability Theory Washington State Univ., April (1972) pp. 179-194.

ARO-26	J. B. Keller	Some Trends in Applied Mechanics Pub: Proceedings of the First Iranian Congress of Civil Engineering & Engineering Mechanics, Pahlavi Univ., Shiraz, Iran, May (1972) pp. 103-124.
ARO-27	J. B. Keller	Nonlinear Stability Theory Pub: Proceed. Conference on Stability Theory Washington State Univ., April (1972) pp. 85-98.
ARO-28	B. Seymour M. Mortell	Nonlinear Resonant Oscillations in Open Tubes Pub: J. Fluid Mech., 60, 4 (1973), pp. 733-749.
ARO-29	R. E. O'Malley J. Chen	On the Asymptotic Solution of a Two-Parameter Boundary Value Problem of Chemical Reactor Theory Pub: SIAM J. Math. Anal., 26, 4 (1974), pp. 717-729.
ARO-30	B. Seymour M. Mortell	Standing Waves in an Open Pipe: A Nonlinear Initial-Boundary Value Problem Pub: Z.A.M.P., 24 (1973), pp. 473-487.
ARO-31	H. Ockendon	Resonant Surface Waves Pub: J. Fluid Mechanics, 59, Part 2 (1973) pp. 397-413.
ARO-32	J. B. Keller	Nonlinear Forced and Free Vibrations in Acoustic Waveguides Pub: J. Acoustical Soc. Amer., 55, 3 (1974), pp. 524-527.
ARO-33	B. Seymour	Nonlinear Geometric Acoustics Pub: Mechanics Today, Vol. 2, S. Nemat-Nasser, editor, Permagon Press (1975), pp. 251-312.
ARO-34	F. Hoppensteadt	Asymptotic Stability in Singular Perturbation Problems II: Problems Having Matched Asymptotic Expansion Solutions Pub: J. Diff. Equations, 15, 3 (1974), pp. 510-521.
ARO-35	F. Hoppensteadt	Asymptotic Stability in Networks with Parasitics Pub: Proceed. 11th Annual Allerton Conference on Circuit & System Theory, 1973), pp. 703-707.

ARO-36	F. Hoppensteadt W. Miranker	Numerical Solution of Differential Equations with Rapidly Changing Solutions Pub: IBM Tech. Report, RC 4792, April 1974, pp. 1-39.
ARO-37	F. Hoppensteadt N. Gordon	Nonlinear Stability Analysis of Static States which Arise Through Bifurcation Pub: Comm. Pure Appl. Math., 28 (1975), pp. 355-373.
ARO-38	F. Hoppensteadt W. Miranker	Differential Equations Having Rapidly Changing Solutions: Analytic Methods for Weakly Nonlinear Systems Pub: J. Diff. Eqns., 22 (1976), pp. 237-249.
ARO-39	L. Ting	Periodic Solutions of Nonlinear Wave Equations in N-dimensional Space Acc: SIAM J. Appl. Math., in press.
ARO-40	F. Hoppensteadt	A Nonlinear Renewal Equation with Periodic and Chaotic Solutions Pub: Amer. Math. Soc., Series SIAM-AMS 10 (1976), pp. 51-60.
ARO-41	F. Hoppensteadt S. Persek	Iterated Averaging Methods for Systems of Ordinary Differential Equations with a Small Parameter Acc: Comm. Pure Appl. Math., in press.
ARO-42	J. B. Keller Alice Whittemore	Quantitative Theories of Carcinogenesis Acc: SIAM Review, in press.
ARO-43	F. Hoppensteadt W. Miranker	Applications of Multi-Time Methods to Pattern Recognition and other Problems Pub: Proc., Journees Math. sur les Perturbation Singulieres, Springer-Verlag, Lecture Notes in Math., 594 (1976), pp. 275-287.

ARO-44	Frank C. Hoppensteadt Donald S. Cohen Robert M. Miura	Slowly Modulated Oscillations in Nonlinear Diffusion Processes Pub: SIAM J. Appl. Math., 33 (1977), pp. 217-229.
ARO-45	J. E. Flaherty F. C. Hoppensteadt	Frequency Entrainment of a Forced Van der Pol Oscillator Acc: Studies in Appl. Math. in press.
ARO-46	R. Burridge J. B. Keller	Peeling, Slipping and Cracking— Some One-Dimensional Free Boundary Problems in Mechanics Acc: SIAM Review, in press.
ARO-47	F. C. Hoppensteadt W. L. Miranker M. Levi	Dynamics of the Josephson Junction Acc: Quart. J. Appl. Math., in press.